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LETTER TO THE EDITOR

Hall effect in the mixed state of moderately clean superconductors

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Abstract. The Hall conductivity in the mixed state of a clean ($l \gg \xi_0$) type-II s-wave superconductor is determined from a microscopic calculation within a quasiclassical approximation. We find that below the superconducting transition the contribution to the transverse conductivity due to dynamical fluctuations of the order parameter is compensated by the modification of the quasiparticle contribution. In this regime the nonlinear behaviour of the Hall angle is governed by the change in the effective quasiparticle scattering rate due to the reduction in the density of states at the Fermi level. The connection with experimental results is discussed.

The Hall effect in the mixed state of type-II superconductors has remained a theoretical puzzle for almost thirty years [1]. The existing phenomenological [2, 3] theories predict that the Hall angle in the flux-flow regime is either identical to that in the normal state [2] or constant [3]. Theories which make use of the time-dependent Ginzburg–Landau equations (TDGL) also find no modification of the Hall conductivity in the superconducting state [1]. These predictions are at variance with the strongly nonlinear behaviour (as a function of magnetic field) found in experiments performed on both low- T_c materials [4, 5] and high- T_c cuprates [4, 6]. For dirty superconductors ($l \ll \xi_0$, where l is the mean free path and ξ_0 is the superconducting coherence length), transport coefficients can be determined from microscopic theory by a straightforward expansion in powers of the order parameter, Δ . The results of such a calculation for the transverse resistivity [7] explain qualitatively the sharp increase in the Hall angle observed in experiment (although, to our knowledge, no systematic comparison has been made), and provide the physical basis for a generalized TDGL approach, in which the relaxation rate is assumed to be complex, rather than purely real, to allow for a modification of the transverse transport coefficients [1, 8]. The small parameter in the expansion of the microscopic equations is proportional to both the order parameter and the mean free path, therefore, it is not small in the clean ($l \gg \xi_0$) limit. In this regime a straightforward expansion is not possible and the TDGL equations are not applicable [9], so that an alternative approach is needed for the calculation of the transverse transport coefficients.

In this letter we give the results of a calculation of the Hall conductivity of a clean s-state superconductor in the mixed state near the upper critical field, H_{c2} , which uses a quasiclassical approximation to the microscopic theory. We made this choice as both the normal state and superconducting properties of the low- T_c compounds are well known, and comparison between theory and experiment is fraught with less ambiguity. The quasiclassical approach [10] has been applied successfully in the past to study transport phenomena in superfluids [11] and superconductors [12] and more recently to

the unconventional superconductors [13]. The central quantity in this method is the single-particle matrix Green's function \widehat{G} integrated over the quasiparticle energy

$$\widehat{g}(s, \mathbf{R}; \omega_n, \omega_{n'}) = \begin{pmatrix} g & -f \\ f^\dagger & \bar{g} \end{pmatrix} = \int \frac{d\xi_p}{\pi} \widehat{G}(p, \mathbf{R}; \omega_n, \omega_{n'}). \quad (1)$$

Here the $\omega_n = 2\pi T(n + \frac{1}{2})$ are fermionic Matsubara frequencies, s is the normalized parametrization of the Fermi surface, \mathbf{R} is the centre of mass coordinate and p is the relative momentum. Since the Green's function is strongly peaked at the Fermi momentum p_f , which is normally far larger than any other momenta in the problem, slower varying quantities such as the self-energy and external potential can be expanded around their values at the Fermi surface. The result of such an expansion [11] in the small parameter $1/\xi_0 k_f \sim \Delta/\epsilon_f$ is a set of transport-like equations for the quasiclassical propagator \widehat{g} . We have generalized these equations to include terms responsible for the Hall effect in a charged superfluid. Technical details of the derivation will be reported elsewhere [14]; here we use the equations to determine the transverse dc conductivity.

We use linear response theory in the vector potential $\mathbf{A}(\omega_0)$ describing a constant electric field $\mathbf{E} = E\hat{x}$. The magnetic field $\mathbf{H} = H\hat{z}$, chosen parallel to the z -axis, is described by the vector potential $\mathcal{A}(\mathbf{R}) = Hx\hat{y}$. We consider a spherical Fermi surface and use the Born approximation for s -wave impurity scattering characterized by a collision rate τ^{-1} . The spatial dependence of the order parameter is modelled by the periodic Abrikosov vortex lattice

$$\Delta(\mathbf{R}) = \sum_{k_y} C_{k_y} e^{ik_y y} \Phi_0(x - \Lambda^2 k_y). \quad (2)$$

Here $\Phi_0(x)$ is the lowest energy eigenfunction of the linearized Ginzburg–Landau equation (i.e. the eigenfunction of a harmonic oscillator with Cooper pair mass $M = 2m$ and frequency ω_c) and $\Lambda^2 = (2eH)^{-1}$ is the magnetic length ($\Lambda \sim \xi_0$ for fields $H \sim H_{c2}$). This approach is appropriate provided that the broadening of the levels in the vortex core is large compared to their spacing $1/\tau \gg \Delta^2/\epsilon_f$; it breaks down in the superclean regime [15]. In the clean limit the finite lifetime is accounted for by replacing ω_n by $\tilde{\omega}_n = \omega_n + (2\tau)^{-1} \langle g(\tilde{\omega}_n) \rangle$ (angled brackets denote an average over the Fermi surface); corrections to the order parameter due to impurity renormalization are of the order $O(\Lambda/l)$ and can be ignored. The equations for the unperturbed functions f and g and the linear, in \mathbf{A} , corrections to the propagator f_1 and g_1 are [14]

$$[2\tilde{\omega}_n + \mathbf{v}_f(\nabla - 2ie\mathcal{A})]f = 2i\Delta g \quad (3)$$

$$[2\tilde{\omega}_n + \mathbf{v}_f(\nabla - 2ie\mathcal{A})]f_1 = ie\mathbf{v}_f \mathbf{A}(f + f(-)) + i\Delta(g_1 - \bar{g}_1) + i\Delta_1(g + g(-)) \quad (4)$$

$$\begin{aligned} \left(i\tilde{\omega}_0 + i\omega_c \frac{\partial}{\partial \phi} \right) (g_1 - \bar{g}_1) &= 2e\mathbf{v}_f \mathbf{A}(g - g(-)) + \Delta_1^*(f - f(-)) \\ &+ \Delta_1 (f^\dagger - f^\dagger(-)) + (2\tau)^{-1} \left(\langle f_1^\dagger \rangle (f - f(-)) + \langle f_1 \rangle (f^\dagger - f^\dagger(-)) \right) \\ &- \frac{i}{2} \left[\frac{\partial \Delta_1^*}{\partial \mathbf{R}} \frac{\partial}{\partial \mathbf{p}_\parallel} (f + f(-)) + \frac{\partial \Delta_1}{\partial \mathbf{R}} \frac{\partial}{\partial \mathbf{p}_\parallel} (f^\dagger + f^\dagger(-)) \right. \\ &\left. + 2 \frac{\partial \Delta^*}{\partial \mathbf{R}} \frac{\partial f_1^\dagger}{\partial \mathbf{p}_\parallel} + 2 \frac{\partial \Delta}{\partial \mathbf{R}} \frac{\partial f_1}{\partial \mathbf{p}_\parallel} \right] \end{aligned} \quad (5)$$

where Δ_1 is the change in the order parameter induced by the electric field. There are corresponding equations for f^\dagger and \bar{g} . In these equations σ_z is the Pauli matrix, the Fermi velocity $\mathbf{v}_f(s) = v(\sin\theta \cos\phi, \sin\theta \sin\phi, \cos\theta)$, \mathbf{p}_\parallel is the component of the momentum

parallel to the Fermi surface, $\omega_c = eH/mc$ is the cyclotron frequency, and the angled brackets denote an average over the Fermi surface. We have used the shorthand notations $\omega_n^- = \omega_n - \omega_0$, $g = g(\tilde{\omega}_n)$ and $g(-) = g(\tilde{\omega}_n^-)$, and have introduced $2\tilde{\Omega}_n = \tilde{\omega}_n + \tilde{\omega}_n^-$ and $\tilde{\omega}_0 = \tilde{\omega}_n - \tilde{\omega}_n^-$. The superconducting order parameter satisfies the usual self-consistency condition $\Delta(\mathbf{R}) = gN(0)\pi T \sum_n \int d^2s f(s, \mathbf{R}; \omega_n, \omega_n)$, where g is the coupling constant and $N(0)$ is the density of states at the Fermi surface. The unperturbed function \tilde{g} obeys the normalization conditions [10] $g + \tilde{g} = 0$ and $g^2 - ff^\dagger = -1$. We chose to write an equation for $g_1 - \tilde{g}_1$ since the transport current is given by [16]

$$\mathbf{j} = \frac{1}{2}\pi eN(0)T \sum_n \int d^2s \mathbf{v}_f(s)(g_1(s, \omega_n) - \tilde{g}_1(s, \omega_n)). \tag{6}$$

The functions f_1, f_1^\dagger also depend on g_1, \tilde{g}_1 in this combination only. In equations (3)–(5) we have omitted terms whose contribution to the conductivity is of order $O(\Lambda/l)$ smaller than that of leading order terms. Equation (3) is the well-known static Eilenberger equation [10] and equations (3)–(5) contain all the terms relevant to the Hall effect for a clean type-II superconductor in the high-field region. It is clear from equation (5) that there are several distinct contributions to $g_1 - \tilde{g}_1$ (and therefore the current). The first, the quasiparticle contribution, depends on the unperturbed function g , while the second, proportional to Δ_1 , is due to the dynamical fluctuations of the order parameter induced by the perturbing electric field. The third term in this equation describes the additional scattering of quasiparticles off these dynamic fluctuations; it has the same origin as the Thompson diagram in the analysis of transport in dirty superconductors [17]. The fourth term describes how as vortices move and are deformed by the transport current, the resulting gradients of the order parameter act as driving forces in the transport-like equations. Finally, as the renormalization of the frequency ω_0 in equation (5) depends on the angular average of the quasiparticle propagator, which changes in the superconducting state, the effective transport mean free path is modified.

To proceed we approximate the diagonal part of the quasiclassical propagator by its spatial average [18]. Since the electromagnetic fields in a superconductor vary over distances of the order of the penetration depth λ , this is a very good approximation in the London limit $\kappa = \lambda/\xi_0 \gg 1$; even for compounds with moderate values of κ it remains valid for a wide field range below H_{c2} . In all of the following g will stand for the averaged distribution function. We now solve equation (3) with the normalization condition to determine the unperturbed functions f and g . We then determine f_1 and Δ_1 to leading order in $(\Lambda\Delta/v)^2 \ll 1$ by solving equation (4) together with the self-consistency condition for the order parameter. To accomplish this program the expression $(2\tilde{\omega}_n + \mathbf{v}_f \cdot (\nabla - 2ie\mathcal{A}))^{-1}\Delta$ has to be evaluated. To do this we exploit the oscillatory character of the Abrikosov solution and introduce raising and lowering operators $a = (\Lambda/\sqrt{2})[\nabla_x + i(\nabla_y - 2ieHx)]$ and $a^\dagger = -(\Lambda/\sqrt{2})[\nabla_x - i(\nabla_y - 2ieHx)]$ obeying the usual bosonic commutation relations $[a, a^\dagger] = 1$. If the ground state equation (2) is denoted by $|0\rangle$, the higher eigenstates (modes of the order parameter) are generated by the standard procedure $a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$. Then to make use of the properties of these operators the operator $\mathbf{v}_f(\nabla - 2ie\mathcal{A})$ can be rewritten as $(v \sin\theta/\sqrt{2}\Lambda)[ae^{-i\phi} - a^\dagger e^{i\phi}]$ and the result of its acting on any state $|n\rangle$ evaluated explicitly [14]. Using this approach we are able to determine the unperturbed functions

$$g = -i \operatorname{sgn}(\omega_n) \left[1 - i\sqrt{\pi} \left(\frac{2\Lambda\Delta}{v \sin\theta} \right)^2 W' \left(\frac{2i\tilde{\omega}_n\Lambda \operatorname{sgn}(\omega_n)}{v \sin\theta} \right) \right]^{-1/2} \tag{7}$$

where Δ is the spatial average of the order parameter and $W(z) = e^{-z^2} \operatorname{erfc}(-iz)$, and

$$f = 2ig \frac{\sqrt{\pi}\Lambda}{v \sin \theta} \sum_{m=0}^{\infty} \frac{1}{\sqrt{m!}} \left(\frac{-i}{\sqrt{2}} \right)^m (\operatorname{sgn}(\omega_n))^{m+1} e^{im\phi} W^{(m)} \left(\frac{2i\tilde{\omega}_n \Lambda \operatorname{sgn}(\omega_n)}{v \sin \theta} \right) |m\rangle. \quad (8)$$

The expression for g reproduces correctly the gapped BCS-like function for quasiparticles travelling parallel to the magnetic field, while describing gapless behaviour in all other directions. A similar expression for g has been obtained by Pesch [19]. Since f is a Fourier series in ϕ , the mode with $m = 0$ will couple to a scalar potential, the mode with $m = 1$ to a transverse potential etc. Then we find $\Delta_1 = (ieA\Lambda\sqrt{2})[(1 - i\tilde{\omega}\tau)^{-1} + \omega_c\tau\tilde{\omega}\tau]|1\rangle$ (here $\tilde{\omega}$ is the real external frequency). Using this value to determine the correction to anomalous propagator f_1 and calculate the current from equation(6) we obtain the longitudinal and transverse conductivities up to order $(\Lambda\Delta/v)^2$.

If we use the usual notation σ_n for the normal state conductivity $\sigma_n = N(0)e^2v^2\tau/3$, the enhancement of the transverse current due to Lorentz force driven fluctuations of the order parameter

$$\sigma_{xy}^{fl} = 6\sigma_n\omega_c\tau(\Lambda\Delta/v)^2 \quad (9)$$

is exactly compensated by the modification of the quasiparticle Hall current due to additional scattering off the vortex lattice

$$\Delta\sigma_{xy}^{qp} = -6\sigma_n\omega_c\tau(\Lambda\Delta/v)^2. \quad (10)$$

Similarly, the positive contribution to the transverse conductivity due to the forces generated by gradient of the excited mode of the order parameter

$$\sigma_{xy}^{gr} = 3\sigma_n(\Delta^2\tau/E_f) = 12\sigma_n\omega_c\tau(\Lambda\Delta/v)^2 \quad (11)$$

is cancelled by the additional scattering introduced by the deformed and moving vortex lattice

$$\sigma_{xy}^{Th} = -12\sigma_n\omega_c\tau(\Lambda\Delta/v)^2. \quad (12)$$

As a result, the behaviour of the transverse conductivity σ_{xy} is determined solely by the effect of the modification of the effective elastic scattering time τ_{eff} on the leading order quasiparticle contribution. For the dc conductivity this change is due to the decrease in the number of states at the Fermi surface available for scattering as the superconducting gap opens. We find, in agreement with the result of Pesch [19] for the density of states, that the increase in the relaxation time is a non-analytic function of the small parameter $(\Lambda\Delta/v)$

$$\tau_{eff}^{-1} = \tau^{-1} \left[1 + 4 \left(\frac{\Lambda\Delta}{v} \right)^2 \log \left(\frac{\Lambda\Delta}{\sqrt{2}v} \right) + 2 \left(\frac{\Lambda\Delta}{v} \right)^2 \right] \quad (13)$$

and, up to order $(\Lambda\Delta/v)^2$, the transverse conductivity is given by

$$\sigma_{xy} = \frac{1}{3}N(0)e^2v^2\tau_{eff}(\omega_c\tau_{eff}) = \sigma_n\omega_c\tau \left[1 + 4 \left(\frac{\Lambda\Delta}{v} \right)^2 \left(\log \left(\frac{2v^2}{\Lambda^2\Delta^2} \right) - 1 \right) \right]. \quad (14)$$

The longitudinal conductivity is obtained in a similar way:

$$\sigma_{xx} = \sigma_n \left[1 + 2 \left(\frac{\Lambda\Delta}{v} \right)^2 \left(\log \left(\frac{2v^2}{\Lambda^2\Delta^2} \right) - 1 \right) \right]. \quad (15)$$

In the high-field region the square of the order parameter is linear in the applied magnetic field and is given by [20]

$$\Delta^2 = \frac{1}{\pi N(0)} \frac{H_{c2} - H}{\beta_A(2\kappa_2^2 - 1)} \left(H_{c2} - \frac{T}{2} \frac{dH_{c2}}{dT} \right). \quad (16)$$

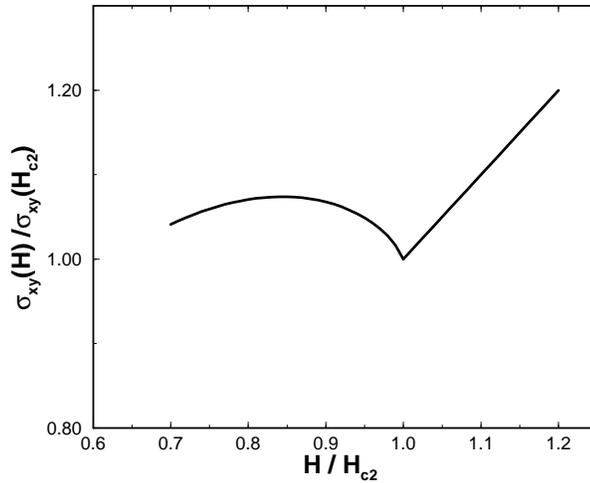


Figure 1. Hall conductivity as a function of the reduced magnetic field.

In figure 1 we show the qualitative behaviour of σ_{xy} as a function of magnetic field, plotted using parameter values for Nb. The transverse conductivity is enhanced below the upper critical field and has negative curvature in the high field region. While the transverse conductivity is proportional to the square of the scattering time, the Hall angle $\tan\theta_H = \sigma_{xy}/\sigma_{xx}$ is only linearly dependent on the scattering time and the corresponding nonlinear dependence on magnetic field is weaker, as can be seen in figure 2. Finally, as the transverse resistivity $\rho_{xy} \approx \sigma_{xy}/\sigma_{xx}^2$ is independent of the effective scattering time, upon entering the superconducting state it remains linear in magnetic field with the same slope as in the normal metal. This behaviour is to be contrasted with that of the Bardeen–Stephen model [2], where the resistivity is modified but the Hall angle obeys the same linear law as in the normal state. On the other hand, the Nozières–Vinen theory [3], which predicts that the Hall angle should be constant in the flux-flow regime below H_{c2} at variance with the result of this work, also finds that the transverse resistivity is identical to that of the normal state, although the individual components of the conductivity tensor are quite different from those found here.

A comparison can be made with the experimental data of Fiory and Serin [5] on high purity Nb. These experiments find a transverse resistivity in the flux-flow regime which is linear in the applied magnetic field over a wide range of fields below H_{c2} . The Hall angle, however, flattens or even increases above its value at H_{c2} before decreasing at lower fields. These results are more suggestive of the behaviour given here than the original interpretation given in terms of the Nozières–Vinen theory. Also, the longitudinal resistivity found in [5] has a distinct increase in slope just below the upper critical field, which is consistent with the logarithmic behaviour given by equation (15). Such comparisons are, of course, only qualitative, and more experimental work is needed to make a detailed comparison with the theory.

To conclude, we give the results of a microscopic calculation of the Hall resistivity of a clean type-II *s*-state superconductor in the high-field limit. We find that the field dependence of the Hall conductivity in the high field regime, which is non-analytic, is entirely due to the change in the density of quasiparticle states at the Fermi level in the superconducting state. At the same time we find that the field dependence of the transverse resistivity below

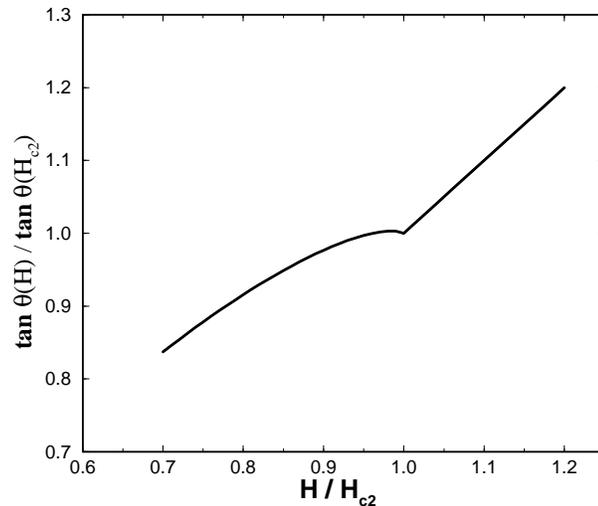


Figure 2. Hall angle as a function of the reduced magnetic field.

the upper critical field remains unchanged. These results are in qualitative agreement with the experimentally observed behaviour. The approach developed here can be generalized to superconductors with unconventional order parameter symmetry; this work is now in progress.

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